Data Collection Activity - Heat It, then Cool It!

1. **The Problem**

A temperature probe is placed in a cup of hot water. It remains there for approximately 40 seconds, then it is removed from this cup and placed in a cup of cold water for another 40 seconds. We will find an algebraic function that models the temperature recorded by the probe over the entire 80 seconds, then answer some questions about the model.

2. **The General Solution (Newton's Law of Cooling/Heating)**

We want to find a function **T(t)** that models the temperature **T** of the probe at any time **t**, measured in seconds. Using a property of physics, called **Newton's Law of Cooling/Heating**, the temperature in an activity such as this can be modeled by an exponential function in the form:

$$T(t) = a \cdot b^t + c \, .$$

3. **The Specific Solution**

First, to find an algebraic function that models our temperature vs. time data, it is clear that we need to write a piecewise function, one rule for the first (approximately) 40 seconds, and another for the last 40 (approximately) seconds. Trace on the data to find the time that separates the two rules. (t= _____ secs)

To find both of these rules, we will use the property of Newton's Law of Cooling.

In order to find a model of the form $T(t) = a \cdot b^t + c$ for the first part of our data, we need to find the constants **a**, **b**, and **c**. We can find the constants **a** and **c** by "tracing" on our scatterplot.

First we will find the constant **c**. According to Newton's Law of Cooling/Heating, and the data collected, the value of **c** would be approximately _____. (Hint: Think about the geometric transformations!)

To find **a**, record the temperature when t=0. (0 , _____)

Substitute this value, with the value of **c**, into our model, and solve for **a**. Show your work below.

a= _____

To find **b**, the last constant in the model, we need another ordered pair. Trace on the data until you get to approximately 10 seconds. Record this ordered pair. (_____ , _____)

Substitute these values into the equation (with the values of **a** and **c**, and solve for the last unknown constant **b**. Show your work below.

b= _____

T(t)= _____

To check your work, graph your equation with your scatterplot to see how it fits the first part of the data.

The equation that fits the second part of the data is similar to the first equation. However, since we are beginning with a time other than t=0, we need to apply a "horizontal shift". Therefore, the resulting form of this function is $T(t) = a \cdot b^{t-h} + c$, where h= _____. Use this form, and the hints given for the first part of the function, and find a rule that fits this part of the data in the scatterplot. Show all of your work on the next page. Again, graph this equation to check it.

a= _____

b= _____

c= _____

T(t)= _____

Finally, combine the results of the two parts of your rule, and write a **piecewise function** that models the temperature vs. time data.

$$T(t) = \begin{cases} \\ \\ \\ \\ \end{cases}$$

4. **Working with the Temperature Function T(t).**

a. Notice on your graph, that there are two times when the temperature is 130 degrees. Use your graph to determine these two times when the temperature is 130 degrees.

t = _____ , t = _____

b. These two t values can also be found algebraically. Using each part of your piecewise function, algebraically find the the times that the temperature is 130 degrees. Compare your answers with your answers determined graphically.

c. The second part of the piecewise function above, is in the form $T(t) = a \cdot b^{t-h} + c$. However, using properties of exponents, this function can be written in the form $T(t) = a \cdot b^{t} + c$. Apply the necesary properties of exponents, and rewrite the second part of your piecewise function, so the exponent on the value of b is just t, and not t-h. (Note: This will not change the values of b or c, but will change your value of a!) When done, graph this function to see if it also fits the data. Show the work below.